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HEAT TRANSFER THROUGH THE AXIALLY SYMMETRIC BOUNDARY LAYER ON A MOVING CIRCULAR FIBRE

D. E. BOURNE* and D. G. ELLISTON†

Department of Applied Mathematics and Computing Science, University of Sheffield, Sheffield

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Abstract—In the process of manufacturing a glass or polymer fibre, a continuous filament of hot material is drawn from an orifice and cools as it passes through the surrounding environment. The rate of heat loss, typified by the local Nusselt number, is of considerable interest from a practical viewpoint.

A simple model of this process is examined wherein the fibre is treated as a continuous infinite circular cylinder issuing steadily from an orifice and penetrating a fluid environment of infinite extent. It is shown that the fluid motion which is generated may be treated as a boundary layer problem. On this basis, and assuming that the fibre is maintained at a uniform temperature, a method is developed for finding the local Nusselt number by means of the Karman–Pohlhausen integral technique. Results are given for several Prandtl numbers (σ) in the range $0 \leqslant \sigma \leqslant 1$. Careful consideration has been given to estimating the probable error arising from the use of the integral method and appropriate correction factors are suggested. A rough averaging procedure allows a comparison to be made with some experimental heat transfer results on fibres at non-uniform temperatures. Satisfactory agreement is obtained.

NOMENCLATURE

a, radius of fibre;
a_1, a_2, a_3, coefficients in series expansion (27);
C_D, local drag coefficient, defined by equation (13);
k, thermal conductivity;
Nu, local Nusselt number, $Q/k(T_w - T_\infty)$;
Q, rate of heat transfer per unit length of fibre;
r, distance from the axis of the fibre;
T, temperature;
T_w, surface temperature of the fibre;
T_∞, ambient temperature of the fluid;
U, speed of the fibre;
u, v, axial and radial fluid velocity components;
x, axial coordinate;
y, distance from the surface of the fibre.

Greek symbols
$\alpha(x)$, parameter in boundary layer velocity profile, equation (7);
$\beta(x)$, parameter in boundary layer temperature profile, equation (19);
γ, Euler's constant;
δ, momentum boundary layer thickness;
δ_T, thermal boundary layer thickness;
ε, dummy variable in equation (9);
κ, thermal diffusivity;
μ, absolute viscosity;
ν, kinematic viscosity;
σ, Prandtl number (ν/κ).

1. INTRODUCTION

IN THE glass and polymer industries, fibres are manufactured by means of a continuous extrusion process. Essentially, a filament of hot

* Senior Lecturer, Department of Applied Mathematics and Computing Science, University of Sheffield.
† Now Assistant Mathematician, United Steels Co. Ltd., Swinden Laboratories, Moorgate, Rotherham.

material is drawn through a circular orifice and wound onto a drum. The rate at which the fibre loses heat as it passes from the orifice to the drum is of considerable practical interest because this has an important bearing on its final characteristics. In particular, glass fibres are believed to derive their remarkably high strength from the high speed of drawing which promotes a rapid rate of cooling (Otto [1] and Bateson [2]). It is with the heat transfer process that we shall be concerned here.

Recently, several authors have given attention to flows generated by continuous moving surfaces. Foremost amongst the investigators was Sakiadis [3–5] who considered the boundary layer flow which develops when an unending flat sheet issues from a slot and moves steadily through a fluid which would otherwise be stationary; the corresponding axially symmetric boundary layer on a circular cylinder issuing steadily from an orifice was also singled out for study. Sakiadis was concerned with calculating the main momentum boundary layer characteristics, such as the drag coefficient. Much of Sakiadis's work on the axially symmetric flow is relevant to the present problem and we shall later explain it in detail. We shall also discuss the probable accuracy which his method of solution achieves.

The problem of heat transfer through boundary layers on continuous moving flat sheets has been studied by Tsou *et al.* [6]. They obtained solutions for uniform wall temperature and for uniform heat flux conditions. Another important contribution has been made by Erickson *et al.* [7] who investigated theoretically the rate of cooling of a flat sheet when it penetrates a fluid environment. Due allowance was made for the heat capacity of the sheet and hence it was not constrained to remain at a uniform temperature.

Theoretical work on heat transfer from a moving glass fibre has been presented by Glicksman [8]. He investigated the dependence of the fibre temperature on distance from the orifice and remarked that the key problem is to determine the Nusselt number. Making the boundary layer approximations, Glicksman derived a formula for the Nusselt number based upon some earlier work by Glauert and Lighthill [9] on axisymmetric laminar boundary layer flow over a *fixed* semi-infinite cylinder. However, Sakiadis [3, 5] demonstrated that there is a fundamental difference between the boundary layer on a moving cylinder (to which the fibre approximates): it was shown that the drag coefficient on a moving cylinder is about 20 per cent less than that on a fixed cylinder. A similar difference can be expected in the corresponding Nusselt numbers.

Glicksman also used Reynolds' analogy to obtain the Nusselt number from Glauert and Lighthill's result for the drag coefficient. However, as noted by Glicksman, Reynolds' analogy is strictly accurate only if the Prandtl number of the fluid is unity. The error in this approximation can be appreciable even for air with a Prandtl number of about 0·72. It should be mentioned that this particular error could have been eliminated by using instead the solution to the problem of heat transfer through the axisymmetrical boundary layer on a fixed cylinder for arbitrary Prandtl number which has been given by Bourne and Davies [10], Bourne *et al.* [11] and by Eshghy and Hornbeck [12].

The object of the present paper is to give a method for calculating the Nusselt number which is free from the disadvantages inherent in Glickman's method: the method deals directly with a moving fibre and is applicable to fluids of arbitrary Prandtl number.

2. FORMULATION OF THE PROBLEM

Since the present study of heat transfer is motivated by a problem of considerable practical interest, it seems desirable to consider first the basic approximations which we shall make in order to understand the extent to which the theory is realistic.

One of the most important underlying assumptions is that we can treat the problem as one of heat transfer through an axially symmetric laminar boundary layer. Typically, the speed of a drawn fibre will be in the range 100–600 cm/s and the length of fibre of interest will be about 50 cms. It follows that for a fibre passing through air or some similar environment, the Reynolds number will be in the range of about 10^4–10^5. This is certainly high enough for the boundary layer approximations to be applicable. The assumption of axial symmetry may not always be realistic since the fibre may undergo transverse vibrations, but provided the oscillations are not too rapid the boundary layer will not be seriously disturbed.

There is some doubt as to the extent to which the boundary layer may be turbulent, but it is well substantiated that for flow over flat plates turbulence occurs at a Reynolds number of about 5×10^5. It seems unlikely that the critical Reynolds number in the present situation will be vastly different from this. Accordingly, we assume that laminar flow conditions prevail.

A further assumption we shall make is that the radius of the fibre is a constant. In practice, the initial radius may be about 10^{-1} cm but this decreases very rapidly to about 2×10^{-3} cm and then remains almost constant. The large change in radius usually occurs within a distance of 1–2 cm from the orifice, which is only 2–4 per cent of the total distance to the drum onto which the fibre is wound. As an approximation, it thus seems entirely reasonable to neglect the variation of the radius near the orifice.

The most serious difficulty is that the temperature of the fibre is not uniform. Typically, the temperature decreases by as much as 1000°C in passing from the orifice to the drum and consequently the physical constants of the environment through which the fibre moves may vary considerably. Problems involving variable fluid properties are notoriously difficult to handle and we can see no way of dealing with this one analytically. Instead, we shall be content to replace the non-uniform fibre temperature by a uniform average value and also neglect any variation of the physical properties of the fluid. Although this procedure may seem a severe shortcoming, the final results are rather surprisingly useful. We shall show, for example that the dependence of the Nusselt number on the drawing speed and fibre radius is supported by some experimental findings of Alderson *et al.* [13]. Further, Glicksman [8] has shown how knowledge of the average Nusselt number can be used to estimate the dependence of fibre temperature on distance from the orifice.

Finally, it should be mentioned that we assume that forced convection is the dominant heat transfer mechanism and also neglect any viscous dissipation of energy in the boundary layer. By examining the orders of magnitude of the Rayleigh and Eckert numbers, it is readily verified that the conditions under which we may neglect natural convection and viscous dissipation are well fulfilled.

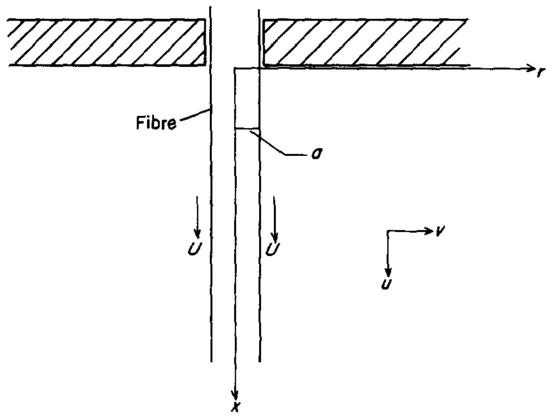

FIG. 1. Endless circular fibre drawn steadily downwards through a circular orifice.

With the approximations described above, the problem to be undertaken is reduced to that of finding the Nusselt number for a continuous circular fibre issuing from an orifice into a homogeneous fluid (Fig. 1). It will be assumed that the fibre is maintained at a uniform temperature T_w and that it moves in the axial direction away from the orifice with constant speed U. At large distances from the fibre, the fluid is at rest and at a uniform temperature T_∞.

We take coordinates x and r which measure distance along the axis of the fibre from the orifice and distance from the axis respectively. Let u, v, respectively, be the axial and radial components of the fluid velocity (Fig. 1) and denote the fluid temperature by T. The appropriate boundary layer equations are

$$r\frac{\partial u}{\partial x} + \frac{\partial}{\partial r}(rv) = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\kappa}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{3}$$

where ν, κ are, respectively, the kinematic viscosity and thermal diffusivity of the fluid. The boundary conditions are

$$u = U, \quad v = 0, \quad T = T_w \quad \text{at} \quad r = a, \tag{4}$$

$$u \to 0, \quad v \to 0, \quad T \to T_\infty \quad \text{as} \quad r \to \infty, \tag{5}$$

where a is the radius of the fibre.

3. CALCULATION OF THE VELOCITY PROFILE

The first step towards finding the rate of heat transfer is to determine the velocity profile from equations (1) and (2). This has been carried out by Sakiadis [5] using the Karman–Pohlhausen technique, and it is convenient to recall here the essentials of the analysis.

Integrating equations (1) and (2) from the surface of the cylinder to infinity leads to the momentum integral equation, viz.

$$\frac{d}{dx}\int_0^\infty u^2(a + y)\,dy = -a\nu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{6}$$

where $y = r - a$ denotes distance from the surface of the cylinder.

Denoting the thickness of the momentum boundary layer by $\delta(x)$, Sakiadis adopted as a suitable Pohlhausen profile

$$\frac{u}{U} = 1 - \frac{1}{\alpha(x)}\log_e\left(1 + \frac{y}{a}\right) \quad \text{for} \quad y \leqslant \delta(x) \tag{7}$$

and

$$\frac{u}{U} = 0 \quad \text{for} \quad y \geqslant \delta(x). \tag{8}$$

This profile has good accuracy near to the fibre, which is known to be a particularly desirable feature when calculating surface characteristics. Furthermore, it is asymptotically of the correct form as $x \to \infty$. For, when $x \to \infty$, the inertia terms in the equation of motion (2) vanish (because u becomes uniform) and the reduced equation is then satisfied by (7) identically.

By forcing the profile to satisfy the momentum integral equation (6), the equation to determine the free parameter $\alpha(x)$, was found to be

$$\frac{2\nu x}{Ua^2} = \lim_{\varepsilon \to 0}\int_\varepsilon^{\alpha(x)}\left(\frac{e^{2t}}{t} - \frac{e^{2t}}{t^2} + \frac{1}{t} + \frac{1}{t^2}\right)dt. \tag{9}$$

Sakiadis evaluated the integral in (9) by a numerical method, but it can be expressed in terms of the tabulated exponential integral function $Ei(z)$, defined by

$$Ei(z) = \int_{-\infty}^{z}\frac{e^t}{t}\,dt. \tag{10}$$

Integrating the second term by parts, the integral on the right-hand side of (9) is found to be

$$\frac{e^{2\alpha}}{\alpha} - \frac{e^{2\varepsilon}}{\varepsilon} - Ei(2\alpha) + Ei(2\varepsilon) + \log_e 2\alpha - \log_e 2\varepsilon - \frac{1}{\alpha} + \frac{1}{\varepsilon}.$$

Using the asymptotic formula

$$Ei(2\varepsilon) \sim \gamma + \log_e(2\varepsilon) \quad \text{as} \quad \varepsilon \to 0, \tag{11}$$

where $\gamma = 0{\cdot}5772\ldots$ is Euler's constant, it follows that

$$\frac{2\nu x}{Ua^2} = \frac{e^{2\alpha} - 1}{\alpha} - Ei(2\alpha) + \log_e(2\alpha) + \gamma - 2. \tag{12}$$

By using the appropriate tables, values of α as a function of $2\nu x/Ua^2$ were determined from (12) and were found to agree with those tabulated by Sakiadis [5].

The Karman–Pohlhausen procedure is not, of course, exact and it will be of some interest later to have available an estimate of the likely error incurred in using it to find the local drag coefficient, defined as

$$C_D = -2\pi a\mu(\partial u/\partial y)_{y=0}/(\mu U), \qquad (13)$$

where μ is the absolute viscosity. Substituting the velocity profile (7), we find

$$C_D = 2\pi/\alpha(x). \qquad (14)$$

Now it is readily deduced from (9) that

$$\frac{2\nu x}{Ua^2} \sim \tfrac{1}{3}\alpha^2 \quad \text{as} \quad \alpha \to 0. \qquad (15)$$

It follows that

$$C_D \sim 0{\cdot}816\pi \left(\frac{Ua^2}{\nu x}\right)^{\frac{1}{2}} \quad \text{as} \quad x \to 0. \qquad (16)$$

But in limit when $x \to 0$, the local drag coefficient on the fibre should be the same as that on a semi-infinite flat sheet issuing from a slot with steady speed U into a fluid at rest at large distances. Sakiadis [4] solved this problem exactly and showed that

$$C_D = 0{\cdot}888\pi \left(\frac{Ua^2}{\nu x}\right)^{\frac{1}{2}}. \qquad (17)$$

The drag coefficient at the leading edge of the fibre is therefore underestimated by about 8 per cent using the Karman–Pohlhausen method. Since the assumed velocity profile becomes increasingly accurate as x increases (and is asymptotically correct), it seems very likely that the error in the drag coefficient will decrease with increasing x.

To obtain an estimate of how the error varies, we may consider the work of Glauert and Lighthill [9] who encountered a similar situation in their Karman–Pohlhausen treatment of the axially symmetric boundary layer on a fixed cylinder. They found that the drag coefficient is underestimated by 13 per cent at the leading edge, and that the error decreases as x increases. Using the formulae given in their paper, we find that the errors are about 12 per cent and 3 per cent when $\log_{10}(\nu x/Ua^2)$ is -3 and 7, respectively. If we assume that the error decreases linearly in this range of $\log_{10}(\nu x/Ua^2)$ and adjust the values of the drag coefficient predicted by the Karman–Pohlhausen solution accordingly, we obtain values which differ by about 1·5 per cent at most from those finally recommended by Glauert and Lighthill.

In the present problem, the exact solution for large values of $\nu x/Ua^2$ is not available and consequently, in this range, it is not easy to estimate the error in the values for the drag coefficient. However, it seems plausible to suppose that the error will decrease in a manner somewhat similar to that in the problem of flow over a fixed cylinder. We shall assume, therefore, that the Karman–Pohlhausen solution underestimates the drag coefficient by 8 per cent at $\log_{10}(\nu x/Ua^2) = -3$ and that this underestimate decreases linearly to 2 per cent at $\log_{10}(\nu x/Ua^2) = 7$. Table 1 displays the correction factor calculated on this basis corresponding to various values of $\log_{10}(\nu x/Ua^2)$. The values of the drag coefficient C_D which we would recommend at these values of $\log_{10}(\nu x/Ua^2)$ may be obtained by multiplying the results under the column $\sigma = 1$ by the appropriate correction factor. [When $\sigma = 1$, the drag coefficient C_D is identical to the Nusselt number Nu defined later, equation (26)].

Finally in this section, we note that in the case of flow over a fixed cylinder, Seban and Bond [14] showed that the flat plate solution is accurate to within 2 per cent when $\log_{10}(\nu x/Ua^2) < -3$. In the present problem, the flat plate solution of Sakiadis [4] probably has a similar range of validity.

4. THE RATE OF HEAT TRANSFER

The Karman–Pohlhausen method proposed for finding the temperature distribution and rate of heat transfer is similar to that used by Bourne *et al.* [11] in the corresponding problem of flow over a fixed cylinder.

Integrating the energy equation (3) from the surface of the cylinder to infinity, we obtain the energy integral equation, viz.

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^\infty u(T - T_\infty)(a + y)\,\mathrm{d}y = -a\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{18}$$

A suitable form for the temperature profile is readily found: for, the profile for T should be essentially similar to that for u [given by equations (7) and (8)] because $T - T_\infty$ and u satisfy similar differential equations and similar boundary conditions. Accordingly, we assume that

$$\frac{T - T_\infty}{T_w - T_\infty} = 1 - \frac{1}{\beta(x)}\log_e\left(1 + \frac{y}{a}\right) \quad \text{for } y \leqslant \delta_T(x) \tag{19}$$

and

$$\frac{T - T_\infty}{T_w - T_\infty} = 0 \text{ for } y \geqslant \delta_T(x), \tag{20}$$

where $\delta_T(x)$ is the thickness of the thermal boundary layer. This form for T satisfies the basic differential equation at the surface of the cylinder ($y = 0$) and the appropriate boundary conditions. Furthermore, like the chosen form for u, the profile will be asymptotically correct when $x \to \infty$ because the convection terms on the left-hand side of the differential equation (3) vanish in the limit, and the right-hand side is reduced to zero identically when (19) and (20) are substituted.

Substituting (7), (8), (19) and (20) into the energy integral equation (18) and assuming that $\delta \leqslant \delta_T$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[e^{2\alpha}\left(\frac{1}{\alpha} - \frac{1}{\beta} + \frac{1}{\alpha\beta}\right) - \left(2 + \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha\beta}\right)\right] = \frac{4\kappa}{a^2 U\beta}. \tag{21}$$

The assumption that $\delta \leqslant \delta_T$ implies that the Prandtl number $\sigma = \nu/\kappa$ is less than or equal to unity. An equation similar to (21) for the case when $\sigma > 1$ can easily be derived, but this range of Prandtl numbers will not be considered here.

The factor $\mathrm{d}\alpha/\mathrm{d}x$ which is implicit in (21) may be eliminated by means of equation (9). Differentiating the latter equation with respect to α yields

$$\frac{\mathrm{d}x}{\mathrm{d}\alpha} = \frac{Ua^2}{2\nu\alpha^2}[e^{2\alpha}(\alpha - 1) + \alpha + 1]. \tag{22}$$

Performing the differentiation in (21) and using (22), it follows after some simplification that

$$\frac{\mathrm{d}\beta}{\mathrm{d}\alpha}[e^{2\alpha}(\alpha - 1) + \alpha + 1] + \beta\alpha^{-1}[e^{2\alpha}(2\alpha\beta - 2\alpha^2 + 2\alpha - \beta - 1) + \beta + 1] = 2\beta\sigma^{-1}\alpha^{-1}[e^{2\alpha}(\alpha - 1) + \alpha + 1]. \tag{23}$$

It can easily be deduced from equations (7) and (8) that $\alpha(x) = \log_e[1 + a^{-1}\delta(x)]$; and similarly from equations (19) and (20) we obtain $\beta(x) = \log_e[1 + a^{-1}\delta_T(x)]$. Since $\delta(0) = \delta_T(0) = 0$, it follows that

$$\beta = 0 \quad \text{when} \quad \alpha = 0. \tag{24}$$

This is the necessary boundary condition for the differential equation (23).

The local rate of heat transfer, per unit length of cylinder is

$$Q(x) = -2\pi ak(\partial T/\partial y)_{y=0}. \tag{25}$$

Defining the Nusselt number Nu to be $Q/[k(T_w - T_\infty)]$, it follows from (19) and (25) that

$$Nu = \frac{2\pi}{\beta}. \tag{26}$$

The function β can be obtained in terms of α by integrating equation (23) subject to the boundary condition (24). In the previous section we showed how α can be found as a function of $\nu x/Ua^2$, and hence we are now in a position to determine the Nusselt number.

5. COMPUTATION OF RESULTS

At the point $\alpha = 0$, $\beta = 0$, the expression for $d\beta/d\alpha$ given by (23) is of an indeterminate form. To find β in the neighbourhood of this point, consider the power series expansion

$$\beta = a_1\alpha + a_2\alpha^2 + a_3\alpha^3 + \ldots. \quad (27)$$

By substituting this series in (23) and comparing coefficients of α, α^2 and α^3 in the expansion of each side, it is found that

$$a_1 = \tfrac{1}{3}(\sigma + 2)/\sigma, \quad (28)$$

$$a_2 = a_1(\sigma - 2a_1\sigma + 1)/(3a_1\sigma - 1), \quad (29)$$

$$a_3 = \frac{2a_2[(3\sigma - 2)^2 - 10]}{45(3\sigma + 2)(\sigma + 1)}. \quad (30)$$

In the range $0 \leqslant \alpha \leqslant 0{\cdot}15$, β was determined for various values of σ using the first three terms of the series (27). The differential equation (23) was then forward integrated, starting at $\alpha = 0{\cdot}15$ and advancing by steps of 0·05 to $\alpha = 10$ (which corresponds to $\nu x/Ua^2 = 1{\cdot}15 \times 10^7$). The fifth order Runge–Kutta procedure was used and the calculation performed with the help of an I.C.T. 1907 computer.

Values of the Nusselt number at various values of $\nu x/Ua^2$ in the range $4{\cdot}31 \times 10^{-4}$–$1{\cdot}15 \times 10^7$ (which is likely to be sufficient for most practical purposes) were determined from (26) for $\sigma = 0{\cdot}12$, 0·24, 0·36, 0·48, 0·72 and 1. They are displayed in Table 1. In Fig. 2 we show how $\log_{10} Nu$ varies with $\log_{10} (\nu x/Ua^2)$ for three typical Prandtl numbers.

The question of the accuracy of the computed results has not been decided with certainty. However, in view of the strong similarity between our approach to this problem and the calculation of the drag coefficient carried out by Sakiadis [5], it seems likely that the errors in the two problems will be comparable. (In the particular case when $\sigma = 1$, the errors must, of course, be identical because the problem of calculating the Nusselt number Nu is then the same as that of calculating the drag coefficient C_D.) Accordingly, we suggest that the calculated values of the Nusselt number displayed in Table 1 should be modified by multiplying them by the correction factors (the derivation of which was explained in Section 3) shown in the final column of the table. Because of the lack of complete information about the correction factors, we have not, however, displayed the modified values.

6. DISCUSSION

It is interesting to observe first how the values of the Nusselt number at corresponding values of $\nu x/Ua^2$ differ for $\sigma = 0{\cdot}72$ and 1. This

Table 1. The Nusselt number for various Prandtl numbers (σ) and the proposed correction factors

$\log_{10} \frac{\nu x}{Ua^2}$	Nusselt number						Correction factor
	$\sigma = 0{\cdot}12$	$\sigma = 0{\cdot}24$	$\sigma = 0{\cdot}36$	$\sigma = 0{\cdot}48$	$\sigma = 0{\cdot}72$	$\sigma = 1$	
−3·3656	21·62	40·82	57·97	73·38	100·1	125·7	0·920
−2·7489	10·95	20·61	29·20	36·91	50·17	62·84	0·921
−1·2265	2·435	4·469	6·213	7·728	10·25	12·57	0·931
−0·4527	1·380	2·460	3·341	4·080	5·252	6·284	0·935
0·5521	0·8644	1·455	1·893	2·234	2·736	3·142	0·941
1·3849	0·6915	1·106	1·388	1·595	1·880	2·095	0·946
2·1837	0·5982	0·9158	1·117	1·257	1·440	1·571	0·951
2·9811	0·5353	0·7894	0·9408	1·042	1·169	1·257	0·956
3·7841	0·4877	0·6969	0·8152	0·8917	0·9851	1·047	0·961
4·5945	0·4495	0·6251	0·7201	0·7799	0·8513	0·8977	0·966
5·4111	0·4176	0·5673	0·6452	0·6933	0·7495	0·7855	0·971
6·2327	0·3904	0·5195	0·5847	0·6242	0·6696	0·6982	0·975
7·0609	0·3667	0·4794	0·5347	0·5676	0·6051	0·6284	0·980

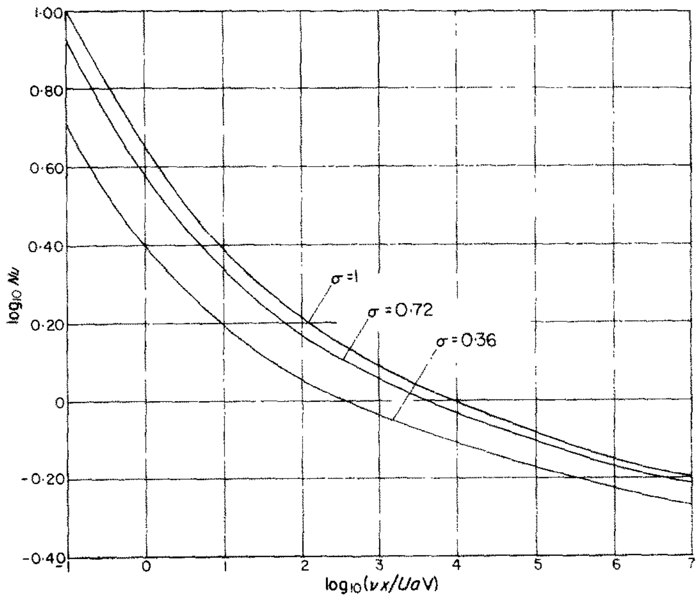

FIG. 2. Logarithm of the local Nusselt number as a function of $\log_{10}(\nu x/Ua^2)$ for three different Prandtl numbers (σ).

gives an indication of the error which arises when the Prandtl number of air is taken to be unity rather than the actual value of about 0·72. The error is found to range from an underestimate of about 26 per cent at small values of $\nu x/Ua^2$ to 4 per cent at large values.

Secondly, it is of interest to compare the theoretical results obtained with those based on an approximate formula devised by Glicksman [8], which we mentioned in the Introduction. In our notation, Glicksman's formula is

$$Nu = \frac{5{\cdot}87}{\log_{10}(4\nu x/Ua^2)} - \frac{3{\cdot}32}{[\log_{10}(4\nu x/Ua^2)]^3}. \tag{31}$$

For fibres drawn through air, Glicksman suggests that the formula should be reasonably accurate when $\nu x/Ua^2 > 10$. However, compared with values obtained from Table 1, we find that it overestimates the Nusselt number by about 37 per cent when $\nu x/Ua^2 = 24$ and by about 27 per cent when $\nu x/Ua^2 = 1{\cdot}15 \times 10^7$. Glicksman's use for air of a Prandtl number of unity is partly responsible for these discrepancies, but the major contribution is a consequence of taking over for the moving fibre Glauert and Lighthill's results [9] for flow over a stationary fibre.

Some experimental work on drawn glass fibres has been carried out by Alderson *et al.* [13]. They determined: (i) the dependence of the average Nusselt number on the fibre radius a when the flow rate πUa^2 is fixed; and (ii) the variation of the average Nusselt number with the speed U when the radius a is fixed. The experiments were carried out on fibres of length 50 cm whose temperature fell from 500°C to 100°C and which passed through air at 20°C. Strictly, the theory developed in this paper is applicable only to fibres at a uniform constant temperature, but by replacing the variable temperatures in the experiments by average values an approximate comparison can be made, as follows.

Assuming that the temperature variations are linear, the mean fibre temperature is 300°C and consequently the mean air temperature is

160°C. The physical constants of air required in the calculation are thus assumed to have the values they take at this temperature. For a flow rate of 10^{-3} cm^3/s, the Nusselt number was calculated at the five points $x = 10, 20, 30, 40$ and 50 cm and the average of these values determined. Since the theory indicates that the Nusselt number is a function of $\nu x/Ua^2$ only, there is, of course no dependence on a for a given flow rate. Figure 3 shows that the theoretical result falls on average about 8 per cent below the experimental values of Alderson *et al.* [13]. In view of the rough averaging procedure we have used, the extent of the agreement is closer than anticipated.

In a similar way, the average Nusselt number was calculated for a fibre of radius $14{\cdot}5 \times 10^{-4}$ cm for several different values of U in the range 100–600 cm/s. Figure 4 shows that there is again reasonably close agreement between the theoretical and experimental values.

We note finally that although we have concerned ourselves with fluids of Prandtl number $\sigma \leqslant 1$, there are essentially no new difficulties in carrying out calculations in the range $\sigma > 1$.

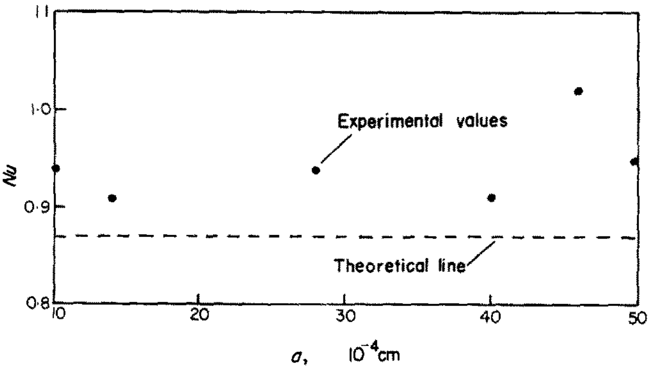

FIG. 3. Comparison of experimental and theoretical values of the average local Nusselt number for variable fibre radius a and a fixed flow rate of 10^{-3} cm^3/s.

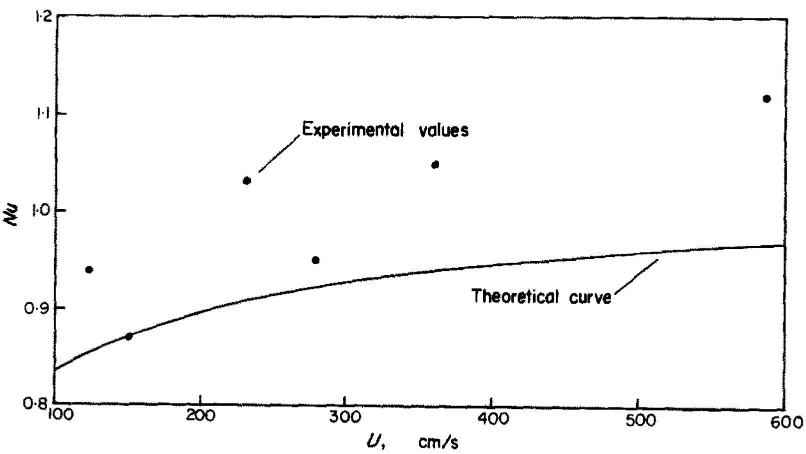

FIG. 4. Comparison of experimental and theoretical values of the average local Nusselt number for variable fibre speed U and fixed radius of $14{\cdot}5 \times 10^{-4}$ cm.

The basic differential equation (23) requires modification because $\delta > \delta_T$ when $\sigma > 1$, but this can easily be accomplished. Otherwise, no changes are necessary.

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TRANSPORT DE CHALEUR À TRAVERS LA COUCHE LIMITE À SYMÉTRIE DE RÉVOLUTION SUR UNE FIBRE CIRCULAIRE EN MOUVEMENT

Résumé—Dans le processus de fabrication d'une fibre de verre ou de polymère, un filament continu de matériau chaud est étiré à partir d'un orifice et se refroidit au fur et à mesure qu'il traverse l'ambiance extérieure. La vitesse de déperdition de chaleur, caractérisée par le nombre de Nusselt local, est d'un intérêt considérable d'un point de vue pratique.

Un modèle simple de ce processus est examiné dans lequel la fibre est traitée comme un cylindre circulaire continu infini sortant à vitesse constante d'un orifice et pénétrant dans un environnement fluide d'étendue infinie. On montre que le mouvement du fluide qui est ainsi produit peut être traité comme un problème de couche limite. Sur cette base, et en supposant que la fibre est maintenue à une température uniforme, une méthode est élaborée pour trouver le nombre de Nusselt local au moyen de la technique intégrale de Kármán–Pohlhausen. Les résultats sont donnés pour plusieurs nombres de Prandtl (σ) dans la gamme $0 \leqslant \sigma \leqslant 1$. On a considéré soigneusement l'estimation de l'erreur probable provenant de l'emploi de la méthode intégrale et des facteurs de correction convenables sont suggérés. Un procédé de moyenne grossier permet de comparer avec quelques résultats expérimentaux de transport de chaleur sur des fibres à températures non uniformes. Un accord satisfaisant est obtenu.

DER WÄRMEDURCHGANG DURCH DIE ACHSIAL-SYMMETRISCHE GRENZSCHICHT AN EINER SICH BEWEGENDEN FASER VON KREISQUERSCHNITT

Zusammenfassung—Bei der Herstellung von Glas- oder Polymerfasern wird kontinuierlich ein Faden aus heissem Material aus einer Düse gezogen und an der Umgebung abgekühlt. Die Wärmeabgabe, abhängig von der lokalen Nusseltzahl, ist von beträchtlichem Interesse für die Praxis.

Ein einfaches Modell dieses Prozesses wird geprüft, wobei die Faser als ein unendlich langer Zylinder von Kreisquerschnitt behandelt wird, der kontinuierlich aus einer Düse kommt und durch ein flüssiges Medium unendlicher Ausdehnung tritt. Es wird gezeigt, dass die entstehende Flüssigkeitsbewegung als Grenzschichtproblem behandelt werden kann. Auf dieser Grundlage und unter der Voraussetzung, dass die Faser eine einheitliche Temperatur hat, wird mit der Kármán–Pohlhausen-Integralmethode ein Verfahren zur Berechnung der lokalen Nusseltzahl entwickelt. Die Ergebnisse sind für verschiedene Prandtl-Zahlen (σ) im Bereich $0 \leqslant \sigma \leqslant 1$ dargestellt. Zur Abschätzung des Fehlers der durch den Gebrauch der Integralmethode entsteht, wurden sorgfältige Untersuchungen angestellt; entsprechende Korrekturfaktoren werden vorgeschlagen. Eine grobe Überschlagsrechnung erlaubt den Vergleich mit einigen experimentellen Ergebnissen der Wärmeübertragung an Fasern mit nicht einheitlicher Temperatur. Man stellt eine zufriedenstellende Übereinstimmung fest.

ТЕПЛООТДАЧА ДВИЖУЩИМСЯ КРУГЛЫМ ВОЛОКНОМ АКСИАЛЬНО-СИММЕТРИЧНОМУ ПОГРАНИЧНОМУ СЛОЮ

Аннотация—При процессе производства стеклянного или полимерного волокна, из отверстия вытягивается непрерывный филамент горячего материала, охлаждающегося во время прохождения по окружающей среде. Скорость потери тепла, здесь относящийся критерии Нуссельта служит типовым примером, представляет большой интерес с практической точки зрения.

Рассматривалась простая модель этого процесса, при чем волокно принималось, как непрерывнй бесконечный круглый цилиндр равномерно выходящий из отверстия и проходящий по бесконечной жидкой среде. Видно, что образующееся движение жидкости можно считать пограничным слоем. На этом основании, и полагая, что температура волокна поддерживается постоянной, разработали способ нахождения относящегося критерия Нуссельта посредством интегрального метода Карман-Полгаузена. Приводим результаты различных критерий Прандтля (σ) в диапазоне $0 \leqslant \sigma \leqslant 1$. Много внимания уделили оценке погрешности, по всей вероятности возникшей, вследствие применения интегрального метода, и предложили соответствующие поправочные коэффициенты. Приблизительная процедура усреднения позволила провести сравнения с несколькими результатами экспериментальной теплопередачи от волокон с непостоянными температурами. При сравнении получили удовлетворительный ответ.